Excel Spreadsheet simulations

These are a couple of spreadsheets that I threw together to demonstrate nolinear resonances, and coupling.

I. One dimensional nonlinear tracking

This simulates a linear ring with one nonlinear thin lens element. It tracks 5 particles for 300 turns and makes two graphs:

- xx'-phase space, and
- x vs turn number. The differential equation considered is

$$\frac{d^2y}{d\theta^2} + Q^2 y = \epsilon x^q \sum_{n=-\infty}^{\infty} \delta(\theta - n2\pi)$$

Warning: Save your work under a different file name.

2nd Warning: Edit only the cells shaded in yellow.

3rd Warning: When the plots seem to disappear, try a smaller value of the coupling, initial conditions, or changing the tune. Look at the scales, you probably have had one or more of the particle trajectories "blow up".

1. Explore what happens with $q = 2$ (a sextupole type nonlearity) as you move the tune close to 1/3.
   1.1 Set the largest initial conditions to $(x_0, x_0') = (4, 0)$. (Values of 1, 2, 3, 3.5, and 4 are good for x_0 with $x_0' = 0$ to start with.)
   1.2 Set $\epsilon = 0.01$, and $q = 2$.
   1.3 Step Q toward 0.33.
      - What happens if $Q = 0.333$?
      - What happens if $Q = 0.34$?
2. Set $Q = 0.251$
   2.1 Step ϵ from 0.01 to 0.13 in steps of 0.01.
3. Explore $Q = 1$.
4. Look for neat distortions for islands with $q = 3$ and 4.

Method of the simulation:

The actual tracking algorithm for a linear turn plus a nonlinear kick is

$$\begin{pmatrix} x_{n+1} \\ x'_{n+1} \end{pmatrix} = \begin{pmatrix} \cos(2\pi Q) & \sin(2\pi Q) \\ -\sin(2\pi Q) & \cos(2\pi Q) \end{pmatrix} \begin{pmatrix} x_n \\ x'_n \end{pmatrix} + \epsilon \begin{pmatrix} 0 \\ x_n^q \end{pmatrix}.$$

II. Two dimensional coupled tracking

Here I have put in three different versions.

1. Two linear harmonic oscillators with linear coupling. (1st sheet called **Linear1**.)

$$\frac{d^2x}{d\theta^2} + Q^2 x = \epsilon y \sum_{n=-\infty}^{\infty} \delta(\theta - n2\pi)$$
$$\frac{d^2y}{d\theta^2} + Q^2 y = \epsilon x \sum_{n=-\infty}^{\infty} \delta(\theta - n2\pi)$$

1.1 Explore what happens around $Q_x - Q_y \simeq 0$
1.2 Now try $Q_x + Q_y \simeq 0$. (Interesting difference, ain't it?)

2. Two linear harmonic oscillators with linear coupling. (2nd sheet called **Linear**.)

$$\frac{d^2x}{d\theta^2} + Q^2 x = \epsilon (y - x) \sum_{n=-\infty}^{\infty} \delta(\theta - n2\pi)$$
$$\frac{d^2y}{d\theta^2} + Q^2 y = \epsilon (x - y) \sum_{n=-\infty}^{\infty} \delta(\theta - n2\pi)$$

Ok, so this is not much different from the previous example.

3. Two linear harmonic oscillators with nonlinear coupling. (3rd sheet called **Nonlin**

$$\frac{d^2x}{d\theta^2} + Q^2 x = \epsilon (y - x)^q \sum_{n=-\infty}^{\infty} \delta(\theta - n2\pi)$$
$$\frac{d^2y}{d\theta^2} + Q^2 y = \epsilon (x - y)^q \sum_{n=-\infty}^{\infty} \delta(\theta - n2\pi)$$

This should liven things up a bit.

4. Two linear harmonic oscillators with nonlinear coupling. (3th sheet called **bb**.)

$$\frac{d^2x}{d\theta^2} + Q^2 x = \epsilon \left[\frac{y - x}{4} - \frac{(y - x)^3}{32} \right] \sum_{n=-\infty}^{\infty} \delta(\theta - n2\pi)$$
$$\frac{d^2y}{d\theta^2} + Q^2 y = \epsilon \left[\frac{x - y}{4} - \frac{(x - y)^3}{32} \right] \sum_{n=-\infty}^{\infty} \delta(\theta - n2\pi)$$

These are the first two term of the beam-beam force.

$$f(x - y) = \frac{1 - \exp[(x - y)^2]}{x}$$